A MULTI-ITEM INVENTORY MODEL FOR COMBAT STORES SHIPS

Charles Floyd Taylor

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

A Multi-Item Inventory Model
for
Combat Stores Ships

by

Charles Floyd Taylor, Jr.

September 1975

Thesis Advisor: A. W. McMasters

Approved for public release; distribution unlimited.

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A Multi-Item Inventory Model for Combat Stores Ships		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis; September 1975
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Charles Floyd Taylor, Jr.		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, CA 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, CA 93940		12. REPORT DATE September 1975
		13. NUMBER OF PAGES 65
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Naval Postgraduate School Monterey, CA 93940		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Inventory theory
Inventory demand distribution
Combat Stores Ships
Multi-Item Inventory Model
Essentiality

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The inventory model currently used by Combat Stores Ships (AFS's) is described and criticized. A simplified procedure for treating essentiality is presented. A multi-item inventory model was developed which minimizes the expected value of essentiality-weighted units short, subject to a constraint on total investment. With only slight modification, the model can be made to minimize the expected value of essentiality-weighted requisitions short.

20. Abstract (continued)

In tests using actual AFS demand data, the proposed model was compared to the current model and found to be markedly superior; specifically, the model was much less expensive to operate (in terms of investment levels) for fixed levels of performance (in terms of essentiality-weighted units short and in terms of line item effectiveness). At the 95% line item effectiveness level, for example, the proposed model required less than one third the investment required by the current model. An important by-product of the analysis was the discovery that AFS inventory demand conforms closely to a mixed Bernoulli/exponential probability distribution.

A Multi-Item Inventory Model for Combat Stores Ships

by

Charles Floyd Taylor, Jr.
Lieutenant, United States Navy
B.S., Stanford University, 1968

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
September 1975

ABSTRACT

The inventory model currently used by Combat Stores ships (AFS's) is described and criticized. A simplified procedure for treating essentiality is presented. A multi-item inventory model was developed which minimizes the expected value of essentiality-weighted units short, subject to a constraint on total investment. With only slight modification, the model can be made to minimize the expected value of essentiality-weighted requisitions short. In tests using actual AFS demand data, the proposed model was compared to the current model and found to be markedly superior; specifically, the model was much less expensive to operate (in terms of investment levels) for fixed levels of performance (in terms of essentiality-weighted units short and in terms of line item effectiveness). At the 95% line item effectiveness level, for example, the proposed model required less than one third the investment required by the current model. An important by-product of the analysis was the discovery that AFS inventory demand conforms closely to a mixed Bernoulli/exponential probability distribution.

TABLE OF CONTENTS

I. INTRODUCTION

A. DESCRIPTION OF THE INVENTORY

The mission of a Combat Stores Ship (AFS) is to resupply U.S. Navy ships at sea with certain essential supplies, specifically subsistence items (dry and refrigerated foodstuffs), ship's store stock, and Fleet Issue Load List (FILL) material. Higher authority determines which items an AFS carries in its inventory; the AFS must, however, determine how much of each item to stock in order to best accomplish its mission. How these quantities should be determined is the subject of this thesis.

Subsistence items comprise considerable bulk but are relatively easy to manage because only a few hundred line items are involved and human food consumption can be predicted fairly accurately. Ship's store stock consists of items such as soap and toothpaste to be sold to crew members and likewise is relatively easy to manage. The third category, FILL material, consists of approximately 11,000 line items of repair parts and consumable material such as transistors, bearings, teletype paper, hydraulic fluid and ball-point pens, to name only a few. The problem of determining the appropriate quantities to stock of each of these 11,000 items in an environment of uncertain demands and limited investment dollars is exceedingly complex and requires careful study.

The Fleet Issue Load List is promulgated by the Fleet Material Support Office (FMSO) in two versions, one for the Pacific Fleet and one for the Atlantic Fleet. The FILL itself consists of a list of items together with a quantity for each which acts as a lower bound on the amount of that item stocked by the AFS. This quantity is referred to as Prepositioned War Reserve Stock (PWRS) and, as the name implies, is intended as a reserve of material to satisfy fleet requirements immediately following the outbreak of a war. Quantities of material, over and above PWRS, carried for the purpose of filling peacetime requirements are referred to as Peacetime Operating Stock (POS). Material held as PWRS may be issued during peacetime by the AFS only when material held as POS has been depleted.

B. CONSTRAINTS

Inventories held by AFS's are financed through the Navy Stock Fund, Special Accounting Class 207. Without going into detail, the effect of this is that an AFS may place orders for stock so long as the total value of material on hand and on order does not exceed a limit imposed by higher authority. This limit is determined on the basis of perceived need and availability of funds and is referred to below as the investment constraint.

Another possible constraint to be considered is storage capacity. In practice this constraint is binding on only about 200 bulky items per AFS (about 2% of the FILL inventory)

and is not a serious problem, particularly in comparison to the investment constraint. It is therefore not considered further in this thesis.

C. AFS OPERATIONS

The Atlantic Fleet AFS's operating in the Mediterranean Sea are normally resupplied by ship from the Naval Supply Center, Norfolk, Virginia. The Pacific Fleet AFS's operating in the western Pacific Ocean normally reload at the U.S. Naval Supply Depot, Subic Bay, Republic of the Philippines (NSD Subic). Atlantic Fleet AFS's thus face relatively long resupply lead times compared to the near-zero lead times which are normally encountered by Pacific Fleet AFS's reloading at NSD Subic. This difference naturally affects planning and operations. For the sake of brevity, then, only Pacific Fleet AFS operations are discussed herein.

The operating schedule of a Pacific Fleet AFS may best be described in terms of operating cycles. An operating cycle begins when the AFS leaves port and proceeds to rendezvous with other fleet units operating at sea. The AFS then conducts a series of operations called underway replenishments, during which material is transferred to other ships. The cycle ends when the AFS returns to port to reload.

Resupply requisitions normally must be hand-carried to the resupply activity (NSD Subic), which effectively restricts the frequency of placing orders to once per cycle, immediately upon arrival at the resupply port. Loading normally takes

four or five days, after which the AFS is ready to begin another cycle. The inventory control problem faced by an AFS is thus to determine how much of each item to load before leaving port in order to best satisfy anticipated demands during the next operating cycle. This must be done without exceeding the investment constraint.

D. INVENTORY MODELS

The inventory model currently being used to deal with this problem is described and discussed in Chapter II. An alternative model for dealing with the problem is proposed and discussed in Chapter III. The two approaches are compared in Chapter IV.

To facilitate the development of the inventory model of Chapter III, the nature of inventory demand is discussed and analyzed in Appendix A. A statistical model for the distribution of demand is proposed and tested. Certain implications of this distribution which relate to the inventory model of Chapter II are presented in Appendix B.

II. THE CURRENT INVENTORY MODEL

A. DESCRIPTION

The inventory model currently used by all AFS's was first developed for use on submarine and destroyer tenders and on repair ships, which all have UNIVAC 1500 computer systems. It was developed by FMSO and was first implemented on board the USS SHENANDOAH (AD-26), a destroyer tender, in late 1968 [1]. When the decision was made to install UNIVAC 1500 computer systems aboard AFS's, a study [2] was conducted to determine an appropriate inventory model. As a result, the decision was made to use the model originally developed for tenders and repair ships. It was first implemented on board an AFS in 1971.

The implemented model was designed to compute two quantities, a requisitioning objective (RO) and a reorder point (RP), for each line item. In theory, a resupply order was to be placed whenever the inventory position (on-hand plus on-order) reached the RP in order to bring the inventory position (IP) back up to the RO. Backorders are not permitted; all orders which cannot be filled during a scheduled underway replenishment become lost sales because it is not usually known when the customer ship will again have an underway replenishment from the same AFS. In practice the inventory position is reviewed periodically rather than continuously, primarily because resupply orders

cannot be placed nor can they arrive while the AFS is at sea. The IP may therefore fall significantly below the RP before an order is filled or even placed.

Each line item in the inventory is classified as a demand-based item (dbi) or a non-demand-based item (non-dbi). The usual criterion for qualification as a dbi is that an item must have been requested by a customer at least twice in the last six months. Once qualified, an item remains demand-based as long as it has been requested at least once in the preceding six months. For non-demand-based items the RO is set to the PWRS quantity and the RP is set to RO-1 or to a percentage of the RO, depending upon the wishes of the AFS. Typically, more than half of an AFS's inventory is non-demand based.

In order for the model to compute the RO and the RP for demand-based-items, various inputs are needed. Average monthly demand (M) is computed from the most recent 24 monthly demand observations, and unit cost (C) is known. In the documentation furnished to users of the model [3], the five principal user-supplied parameters are described in the following terms:

SL — Safety level factor; the number of months of demand to be included in the safety level. The safety level is intended as a buffer to reduce the fraction of times an item will be out of stock.

OST — Order and shipping time factor; the number of months'demand authorized to be held to cover demands during lead time.

OLM Operating level multiplier; "a management control which is used to mathematically express the availability of investment dollars and the ability to cope with workload produced by resupply orders." Graphs based on FMSO simulations are available to the user to assist in setting this and other parameters [3, 4].

MINQ Minimum months in operating level; "a minimum constraint (lower limit) on the size of the computed operating level. It represents desired months of supply at the average monthly demand rate."

MAXQ Maximum months in operating level; same as MINQ except that it acts as a maximum rather than a minimum constraint.

Using these parameters the model makes the following calculations for each demand-based item in the inventory:

$$RP = \max \begin{cases} PWRS, \\ (SL + OST) \cdot M; \end{cases}$$

$$OL = OLM \cdot \sqrt{M/C} \;;$$

$$RO = RP + OL;$$

where OL is called the operating level and is defined as an economic order quantity [3]. The operating level is constrained as follows:

$$MINQ \cdot M \leq OL \leq MAXQ \cdot M.$$

The relationship between the various quantities is summarized in Figure 1, which has been adapted from [4].

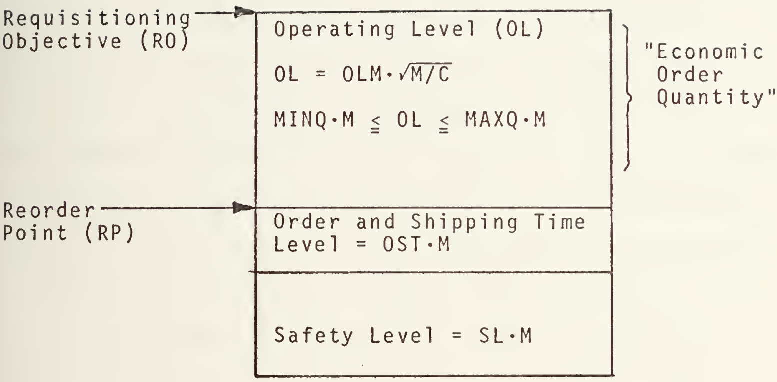

Figure 1. Structure of the current model (when PWRS<RP).

Although much of the documentation of the development of the above model was not available at the time of this writing, discussions with some of the principals involved in the development indicated that, because it was first developed for use by tenders and repair ships, it is similar to the model previously used by those ships. The difference is in the computation of the operating level. The operating level in the earlier model was taken to be a constant times the average monthly demand. The requisitioning objective therefore represented a fixed number of months' demand for every demand-based item in the inventory. This was probably a carry-over from the way funding decisions were (and often still are) made (i.e., a ship's investment constraint was a constant times average monthly sales in dollars).

In an attempt to improve this model, a concept called Variable Operating Level (VOL) was introduced. Under this concept some items were to be stocked in greater quantities and others in lesser quantities, with the constraint imposed on an overall rather than an item-by-item basis. The formula selected for determining the various OL quantities was the classic Wilson economic order quantity, with constraints imposed as described above. A complete development and description of the Wilson economic order quantity formula may be found in Chapter 2 of Hadley and Whitin [5]. The resulting VOL models appears to be a version of the lot-size reorder point model of Chapter 4 of Hadley and Whitin [5] in

which the optimal order quantity (lot size) is approximated by the Wilson order quantity and the reorder point is computed as a constant times average monthly demand.

B. DISCUSSION

There are two main problems inherent in applying this model to the AFS inventory problem. First, it implicitly requires a continuous review system (i.e., a system in which an order is placed as soon as the RP is reached). Usually, an AFS places orders at only one time during the operating cycle, as pointed out in the introduction. This means that frequently orders are not placed until the inventory position is well below the RP. In other words, a continuous-review model is being used in a periodic-review application.

Second, the VOL model appears to be basically a single-item model. The distinguishing feature of this type of model is that each item in the inventory is treated independently of every other item; no consideration is made of any interaction between items. When an investment constraint is present, however, all items interact because of competition for investment dollars. The VOL model itself does not explicitly consider the existence of an investment constraint; in practice the constraint must be arbitrarily imposed by varying the various parameters of the model on a trial-and-error basis until the constraint is satisfied.

One of the fundamental concepts in inventory theory is the notion of stockout risk. A stockout occurs when a demand is received for more than the on-hand quantity of an item. Stockout risk for a given item is the probability that one or more stockouts will occur during a given period (the probability that demands exceed available supply). Stockout risk, often referred to simply as risk, depends on the statistical distribution of demand. For this reason the nature of the distribution of demand is examined in Appendix A.

The usual procedure in a lot-size reorder point model is to set reorder points so as to control stockout risks according to some policy. The policy in the VOL model, as mentioned above, is to set RP's to a fixed number times the average monthly demand for each item. The implications of this policy in terms of stockout risks are explored in Appendix B in light of the results of Appendix A. It is not clear in what sense, if any, this policy is optimal.

In this chapter the VOL inventory model currently used by AFS's has been presented and discussed. Several deficiencies have been pointed out. The inventory model presented in the next chapter is an attempt to remedy these deficiences.

III. THE PROPOSED INVENTORY MODEL

A. DEVELOPMENT

In the introduction it was stated that the inventory control problem faced by an AFS is to determine how much of each item to stock in order to best satisfy anticipated demands without exceeding an investment constraint. The notion of "best satisfying anticipated demands" is, however, quite nebulous. An appropriate measure of effectiveness must be defined.

There are several performance measures in common use. These include line items short, line item effectiveness, units short, requisitions short, and requisition effectiveness. Line items short is defined as the number of different line items for which demands exceed supply in a given period. Line item effectiveness is defined as one minus the ratio of line items short to total line items demanded. Units short, or simply shortages, is the total number (over all items) of units demanded minus the total number of units issued. Requisitions short is defined as the total number (over all items) of customer requisitions which were not satisfied. Requisition effectiveness is one minus the ratio of requisitions short to total requisitions received.

To clarify these definitions and to emphasize the differences between them, a hypothetical example has been constructed where the inventory consists of two items, A and B. There are five units of item A on hand and one unit of item B. Two requisitions are received for each item, the first for one unit and the second for two units. The situation is summarized and the computations of the various performance measures are illustrated in Table 1.

Table 1. Illustration of Performance Measures

Item	On Hand	Requisition Quantities	Requisitions Short	Line Items Short	Units Short
A	5	1,2	0,0	0	0
B	1	1,2	0,1	1	2
Totals			1	1	2

Line item effectiveness $= 1 - \frac{1}{2} = .50$

Requisition effectiveness $= 1 - \frac{1}{4} = .75$

The measures described above treat all items as if they are of equal importance. It seems intuitively appealing that some items would have higher essentiality than others; that is, their non-availability would be more disruptive to fleet operations. For example, a repair part for a ship's radar certainly has higher essentiality than a box of paper clips.

To this author's knowledge, no one has yet devised a practical method for assigning individual shortage costs to each item in an inventory when thousands of items are involved, although many shipboard supply officers already implicitly assign differing levels of essentiality by maintaining what is commonly known as a "Never-Out List." Each supply officer has his own ideas about what should be included on this list based on his own experience and on input from others; however, the list most often includes items whose non-availability would cause the most disruption to fleet operations. Items on this list receive special management attention and are usually stocked in larger quantities than would otherwise be the case in an attempt to reduce stockout risk. The "Never-Out List" implicitly divides the inventory into two categories according to essentiality: those items on the "Never-Out List" may be thought of as "high-essentiality" items, and all other items as "non-high-essentiality" items.

A formalization of this procedure seems appropriate as a first step in dealing with the problem of essentiality. Non-high-essentiality items could be assigned a shortage cost of, say 1.0, while high-essentiality items could be assigned some higher shortage cost. These shortage costs could then be used as weighting factors in the construction of a performance measure which takes into account essentiality.

One such measure is the sum of essentiality-weighted shortages. As an example of how this would be computed, suppose that in a hypothetical inventory a shortage of two units occurs for each of two line items. Suppose that one of the items is a non-high-essentiality item with associated shortage cost 1.0, and that the other is a high-essentiality item with associated shortage cost 10.0. The sum of essentiality-weighted shortages would be computed as follows: 2 x 1.0 + 2 x 10.0 = 22.0. It should be noted that when the shortage cost of every item in the inventory is 1.0, essentiality-weighted shortages coincide with units short.

The objective of the inventory model to be presented below is the minimization of the expected number of essentiality-weighted shortages. This minimization must be done, as discussed in the introduction, in the presence of an investment constraint.

Demand over an operating cycle is assumed to be distributed according to the Bernoulli/exponential distribution (see Appendix A). The demand density of the i^{th} item is then

$$f_i(x) = \begin{cases} 1 - p_i & , x = 0 \\ p_i \lambda_i e^{-\lambda_i x} & , x > 0. \end{cases} \qquad (1)$$

Suppose that the inventory consists of N items, that the i^{th} item has unit purchase cost C_i and unit shortage cost S_i, and that the investment constraint is B dollars. Let R_i be the amount of the i^{th} item to be stocked. The total expected value of essentiality-weighted shortages is given by

$$\sum_{i=1}^{N} S_i \int_{R_i}^{\infty} (x - R_i) f_i(x) dx, \tag{2}$$

and the total investment constraint by

$$\sum_{i=1}^{N} C_i R_i \leqq B. \tag{3}$$

The problem is to find the values $R_i \geqq 0$, $i = 1,2,\ldots,N$, which minimizes expression (2) while satisfying inequality (3).

It should be noted that in the absence of a constraint, expected shortages would be minimized by carrying an infinite amount of each item. This necessarily implies that the constraint will be binding in all cases. The objective function (2) can easily be shown to be convex in R_i. The constraint (3) is, of course, linear. These conditions are sufficient to ensure that a unique optimal solution R_i^*, $i = 1,2,\ldots,N$, exists and can be found using the method of Lagrange.

Let the Lagrange function be denoted by L and the Lagrange multiplier by θ. Then

$$L = \sum_{i=1}^{N} S_i \int_{R_i}^{\infty} (x - R_i) f_i(x) dx + \theta\left(\sum_{i=1}^{N} C_i R_i - B\right). \quad (4)$$

Taking the N + 1 partial derivatives with respect to the decision variables R_i and θ and setting them to zero yields the following N + 1 equations:

$$\bar{F}(R_i) = \theta C_i / S_i, \; i = 1,2,\ldots,N; \quad (5)$$

$$\sum_{i=1}^{N} C_i R_i = B, \quad (6)$$

where $\bar{F}(R_i)$ is the probability that demand exceeds R_i and is therefore the risk associated with stocking R_i units of item i.

The general solution procedure is to first select a positive value for θ. Then, for each item in the inventory, compute the optimal value of risk, abbreviated here as r_i, according to the formula

$$r_i = \theta C_i / S_i. \quad (7)$$

Because some items in the FILL actually have an assigned cost of zero [6], it is necessary to impose a positive lower bound on r_i to ensure the existence of a realistic solution (zero cost would otherwise imply zero risk, which is impossible to achieve with an R_i value less than infinity). This bound will be called MINR and will be assumed to be independent of i. In addition, it is clear that risk can

never be greater than 1.0. It is also true, as shown in Appendix B, that risk for a given line item is bounded above by p_i, the probability that demand for that item will be greater than zero. In addition, many inventory managers may not be willing to tolerate stockout risks greater than some limit, say 0.5, no matter what. This threshold will be called MAXR and will be assumed to be independent of i. An upper bound on r_i should therefore be imposed. For a given item, this upper bound will be the smaller of p_i and MAXR. In summary, r_i should be determined from equation (8).

$$r_i = \begin{cases} \text{MINR if } \theta C_i/S_i < \text{MINR}; \\ \theta C_i/S_i \text{ if MINR} \leq \theta C_i/S_i \leq \min\,(p_i, \text{MAXR}); \\ \min(p_i, \text{MAXR}) \text{ if } \theta C_i/S_i > \min(p_i, \text{MAXR}). \end{cases} \tag{8}$$

Once the value of r_i has been computed for an item, the next step is to find the value of R_i which corresponds to r_i. Using the expression for risk given by equation (A-2) of Appendix A,

$$r_i = p_i e^{-\lambda_i R_i}. \tag{9}$$

It follows that

$$R_i = -(1/\lambda)\ln(r_i/p_i). \tag{10}$$

After computing R_i for each item in the inventory, the final step is to compute the investment level $\sum_{i=1}^{N} C_i R_i$. If the investment level is less than B (the investment constraint), a smaller value of θ should be selected and the process repeated. If it exceeds B, a larger value of θ should be selected. The process ends when the investment level is sufficiently close to B. This may require several iterations the first time the model is used on a given AFS, but after that the optimal value of θ should vary little for each successive use of the model, unless there is a major change in the investment constraint or in the level of demand.

The associated decision rule for the placing of resupply orders is as follows: If, upon periodic review, the inventory position of an item is less than its calculated R_i value, an order should be placed to bring the inventory position of that item up to R_i.

The above model with the expected value of essentiality-weighted shortages as the measure of effectiveness will be referred to subsequently as the EWS model. In the following chapter this model will be compared to the VOL model discussed in Chapter II.

B. DISCUSSION

1. Theoretical Considerations

The inventory model presented above actually belongs to the general class of multi-item, single-period models discussed in Chapter 6 of Hadley and Whitin [5]. A multi-item model is appropriate because it considers the interaction between items implied by the existence of an investment constraint, as the VOL model does not. A single-period model is appropriate for two reasons. First, it considers the fact that an AFS may not order or receive resupply material once it has left port to begin an operating cycle. Second, an AFS is permitted to periodically transfer excess material to an ashore supply activity with full reimbursement, should this become necessary, in order to free investment dollars for other material. It is therefore unnecessary to consider the more complex class of dynamic multi-period models.

Let the length of the operating period be denoted by T and lead time as τ; assume both are known and constant. Following section 5-2 of Hadley and Whitin [5], $f_i(x)$ in equation (1) is assumed to be the density of demand associated with a period of length $T + \tau$. In the case of Pacific Fleet AFS's, the lead time τ does not overlap an operating period; it is normally spent in port. The demands experienced during this time are negligible. For practical purposes, then, only the period T is relevant. For this model to be

applied to Atlantic Fleet AFS's, however, explicit consideration would have to be given to demands during lead time.

2. Shortage Costs

In the presentation of the model it was stated that high-essentiality items should be assigned "some higher shortage cost." The question of how this higher cost should be determined must be addressed. The high-essentiality shortage cost, denoted here as S', is used in order to reduce stockout risk for high-essentiality items. Since there is no obvious a priori means of obtaining the value of S', it is convenient to infer its value from the value of stockout risk desired by the manager of the system for high-essentiality items, assuming his judgement to be optimal. As a first estimate of S', Figure 6 of Chapter IV and the accompanying discussion should prove useful. If it is desired to refine the estimate of the S' after optimal θ is obtained, the following procedure can be used: Suppose that U is an upper bound on the desired level of stockout risk for high-essentiality items. Suppose further that the most expensive high-essentiality item in the inventory has unit cost C'. Using equation (7), the value of S' may then be imputed as follows:

$$S' = \theta C'/U.$$

It may be seen from equation (8) that all lower-cost high-essentiality items will then have lower (or equal) stockout risks assigned.

The parameter MAXR was introduced as a management control on the assignment of risk. When MAXR is binding, the effect is to impute a higher shortage cost for that item in order to reduce risk to MAXR. Care must be exercised in the assignment of MAXR, if too small a value is used, it is conceivable that the investment level might exceed the investment constraint (3) for all values of θ. The sensitivity of the model to changes in the value of MAXR is also examined in the next chapter.

3. <u>The Investment Constraint</u>

The presence of an investment constraint implies through equation (5) that if two items have the same shortage cost, the manager of the inventory system should be willing to accept a higher level of risk for the higher cost item than for the lower cost item. If this is not acceptable, then the higher cost item should be assigned a higher shortage cost.

For the same reasons it is sometimes optimal not to stock a particular item at all (i.e., set $R_i = 0$). This occurs when the optimal risk computed using the model equals p_i, the probability of a non-zero demand. This would be most likely for high-cost, low-essentiality items with a low probability of a non-zero demand. If this is not acceptable in the case of a particular item, the shortage cost for that item should be increased. If it is desired to reduce the

frequency of this phenomenon across the entire inventory, the value of MAXR can be reduced, thus implying actual higher shortage costs for items which would otherwise not be stocked.

4. PWRS

An explicit provision for a PWRS constraint has not been made in this model. One way to apply this constraint would be to set R_i to the value computed by equation (10) or to the PWRS quantity, whichever was larger. This topic is left as an area for further study.

5. Measure of Effectiveness

The above model was designed to minimize the expected value of essentiality-weighted shortages. Only slight changes are necessary, however, in order to optimize on a different measure of effectiveness. If it is desired to minimize the expected value of essentiality-weighted requisitions short, and if the average requisition size for item i is given by Z_i, it is easily shown that the only change necessary is to replace the expression $\theta C_i/S_i$ by $\theta C_i Z_i/S_i$ in equations (5), (7), and (8). The procedure is otherwise identical.

If essentiality-weighting is not desired, it is only necessary to set $S_i = 1.0$ for all i.

IV. EVALUATION AND COMPARISON OF MODELS

A. TESTING PROCEDURE

A computer program was written to evaluate and compare the performance of the VOL and EWS inventory models described above. The program itself is listed following Appendix B; a brief description of its operation is given here.

Actual AFS demand history records for 2758 line items were used as input to the program. The source of these records is discussed in Appendix A. The records contained 24 monthly demand observations for each line item, but no information about unit prices or essentiality. Rather than manually looking up each individual price, the empirical distribution of prices computed by FMSO [6] for the 1975 FILL was used to assign prices at random to individual items. The distribution of prices for the sample items thus approximates the actual distribution of prices for FILL items. At the assigned prices, the value of average monthly demand was about $3.815 million. This figure is probably higher than it should be, judging from the author's experience (two years as Control Officer of AFS-4). No information was available, however, concerning the covariance structure between price and demand, so it is suspected that a number of fast-moving items were assigned higher prices than they should have been. It is not felt that the relative performances of the two models were affected by this difference.

Approximately 10% (278) of the items were randomly designated as high-essentiality items. This figure is not unreasonable, judging again from the author's experience.

Twenty-four months of AFS operations were simulated using each of the two inventory models. As mentioned above, actual demand data was used. Prior to the first period of simulated operations, the values of RO, RP, and R were computed for each item according to the procedures described above. The on hand balance for each item was initialized to the value of RO computed for that item for testing the VOL model and to the value of R computed for that item for testing the EWS model. During each simulated period, the actual demand for that item was compared to the simulated on hand balances for each model. The number of shortages, if any, was computed and performance statistics were accumulated for each model. Simulated on hand balances were then adjusted for each model to reflect issues made. At the end of each period, a reorder review was made. For the VOL model, the simulated on hand balance was compared to the RP; if it was found to be less than or equal to the RP, stock was replenished up to the RO. For the EWS model, stock was replenished whenever the on hand balance was found to be less than R. The ending (simulated) on hand balance for each model for one period then became the beginning balance for the next period. The values of RO, RP, and R were computed only once, prior to the first simulated period.

B. TEST RESULTS

The VOL model has been extensively tested elsewhere [3, 4] and was found to be relatively insensitive to changes in the parameters OLM, MINQ, and MAXQ. Therefore these parameters were fixed at recommended levels [3] and the safety level parameter SL varied in order to evaluate the performance of the VOL model using these particular demand records. The behavior of the model conformed closely to that reported in [3, 4]. Because essentiality-weighted shortages was used as the primary performance measure (the expected values were used in the objective function), tests were run with the high-essentiality shortage cost S' set at 100 and at 1; the former results are given in Table 2 and the latter in Table 3.

Tests of the EWS model were also conducted for some cases with S' = 100 and S' = 1. The parameter θ was varied in order to obtain results at various levels of investment comparable to those obtained in testing the VOL model. The results are given in Tables 4 and 5.

In order to provide alternate performance measures for comparing the two models, the following measures were computed in addition to essentiality-weighted shortages: line item effectiveness for all items, and line item effectiveness for high-essentiality items only.

Table 2. Performance of the VOL model with S' = 100.

SL	INV	EWS	LIE	LIEHE
0.00	.230	233.37	.8049	.8011
0.50	2.138	215.52	.8460	.8462
1.00	4.045	190.62	.8757	.8778
1.50	5.952	174.75	.8948	.9000
2.00	7.860	153.43	.9151	.9185
2.25	8.814	146.22	.9223	.9240
2.50	9.768	141.05	.9286	.9297
2.75	10.722	136.89	.9338	.9348
3.00	11.676	130.26	.9386	.9393
3.25	12.629	120.80	.9429	.9438
3.50	13.583	117.69	.9465	.9472
4.00	15.491	101.32	.9536	.9543
4.50	17.398	95.34	.9596	.9603
5.00	19.306	91.22	.9644	.9649
6.00	23.121	79.82	.9719	.9723
7.00	26.935	72.35	.9776	.9775
8.00	30.747	64.48	.9819	.9823
10.00	38.377	47.18	.9882	.9885

Average Number of Resupply Orders per Month: 538.17

SL: Safety Level factor
LIE: Line Item Effectiveness for all items
LIEHE: Line Item Effectiveness for High-Essentiality items
EWS: Essentiality-Weighted Shortages per item per month
INV: Investment in millions of dollars

Table 3. Performance of the VOL model with S' = 1.

SL	INV	EWS	LIE	LIEHE
0.00	.230	31.52	.8049	.8011
1.00	4.045	27.17	.8757	.8778
2.00	7.860	22.39	.9151	.9185
3.00	11.676	18.34	.9386	.9393
4.00	15.491	15.25	.9536	.9543
5.00	19.306	13.06	.9644	.9649
7.50	28.843	9.79	.9800	.9807
10.00	38.377	8.12	.9882	.9885

Average Number of Resupply Orders per Month: 538.17

Table 4. Performance of the EWS model with S' = 100.

θ	INV	EWS	LIE	LIEHE	R/M
1.0	1.282	114.82	.7490	.9222	491.58
0.1	1.623	65.47	.8351	.9748	681.25
0.02	2.214	54.21	.9031	.9846	811.38
0.01	2.854	51.06	.9258	.9849	851.21
0.008	3.167	50.36	.9327	.9849	866.08
0.006	3.600	49.03	.9419	.9852	885.08
0.005	3.914	46.48	.9475	.9855	895.04
0.004	4.347	45.76	.9539	.9862	901.83
0.002	6.043	44.35	.9679	.9873	916.33
0.001	8.471	42.70	.9767	.9883	928.21
0.0001	13.446	41.28	.9876	.9889	934.25
0.00001	23.628	41.10	.9896	.9889	937.50
0.000001	31.981	41.06	.9899	.9889	937.50
0.0000001	31.981	41.06	.9899	.9889	937.50

MINR: .01
MAXR: .50
R/M: Resupply orders per month

Table 5. Performance of the EWS model with S' = 1.

θ	INV	EWS	LIE	LIEHE	R/M
0.1	1.308	23.34	.8196	.8174	654.96
0.02	1.733	16.88	.8956	.8964	799.08
0.01	2.381	13.54	.9219	.9252	842.88
0.0075	2.819	12.41	.9322	.9350	865.13
0.005	3.522	8.92	.9468	.9472	889.79
0.0025	5.044	6.91	.9659	.9660	909.17
0.001	7.331	4.78	.9811	.9801	925.00
0.0001	14.338	2.93	.9953	.9942	933.33

MINR: .001
MAXR: .50

To facilitate comparison of the test results given in Tables 2 through 5, several graphs were constructed. Figure 2 shows the relative investment levels required in order for each of the two models to provide various levels of effectiveness in units of essentiality-weighted shortages per item per month, with S' = 100. The same comparison is made in Figure 3 with S' = 1. In both cases the differences between the performances of the two models are striking; the EWS model required a much smaller investment for a given level of performance. Looking at it from another point of view, for a given investment the EWS model provided clearly superior performance (i.e., a much smaller value of essentiality-weighted shortages).

Figures 4 and 5 compare the two models with respect to line item effectiveness for all items. In Figure 4, S' = 100 was used, while in Figure 5, S' = 1. Again, the differences are striking. True, the graphs in Figures 4 and 5 cross at about 0.83 line item effectiveness, but the average line item effectiveness was not the objective function; it is included here only for comparison. At realistic investment levels (i.e., above the level of $3.815 million corresponding to the average monthly demand of the data), the performance of the EWS model was much superior, whether judged in terms of essentiality-weighted shortages or line item effectiveness. Specifically, for a given level of line item effectiveness or of essentiality-weighted shortages, the EWS model required

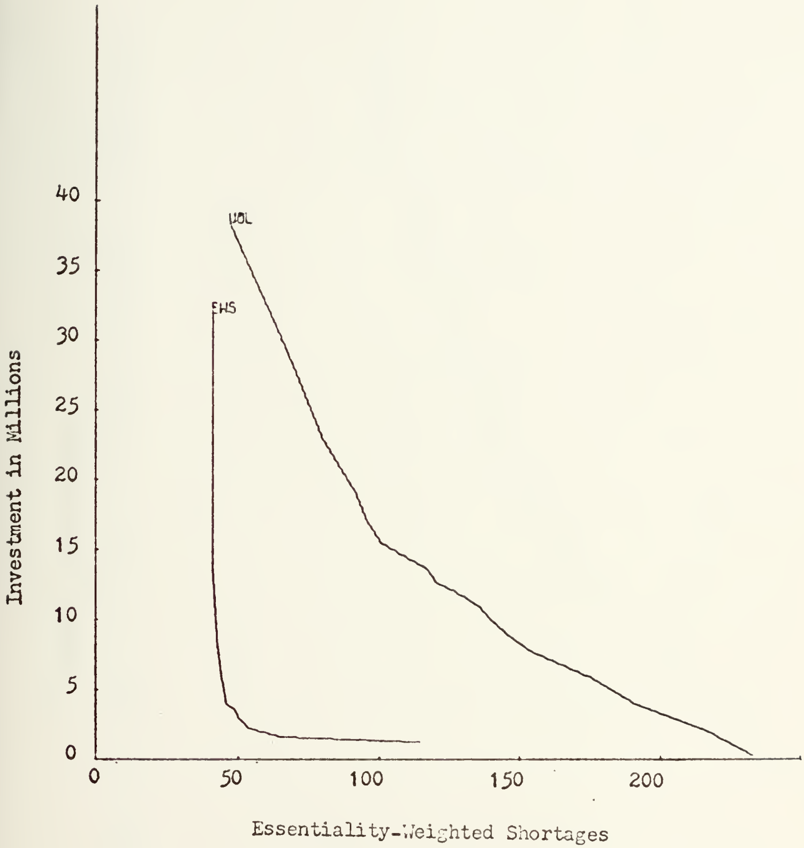

Figure 2: Investment versus Essentiality-Weighted Shortages with S' = 100

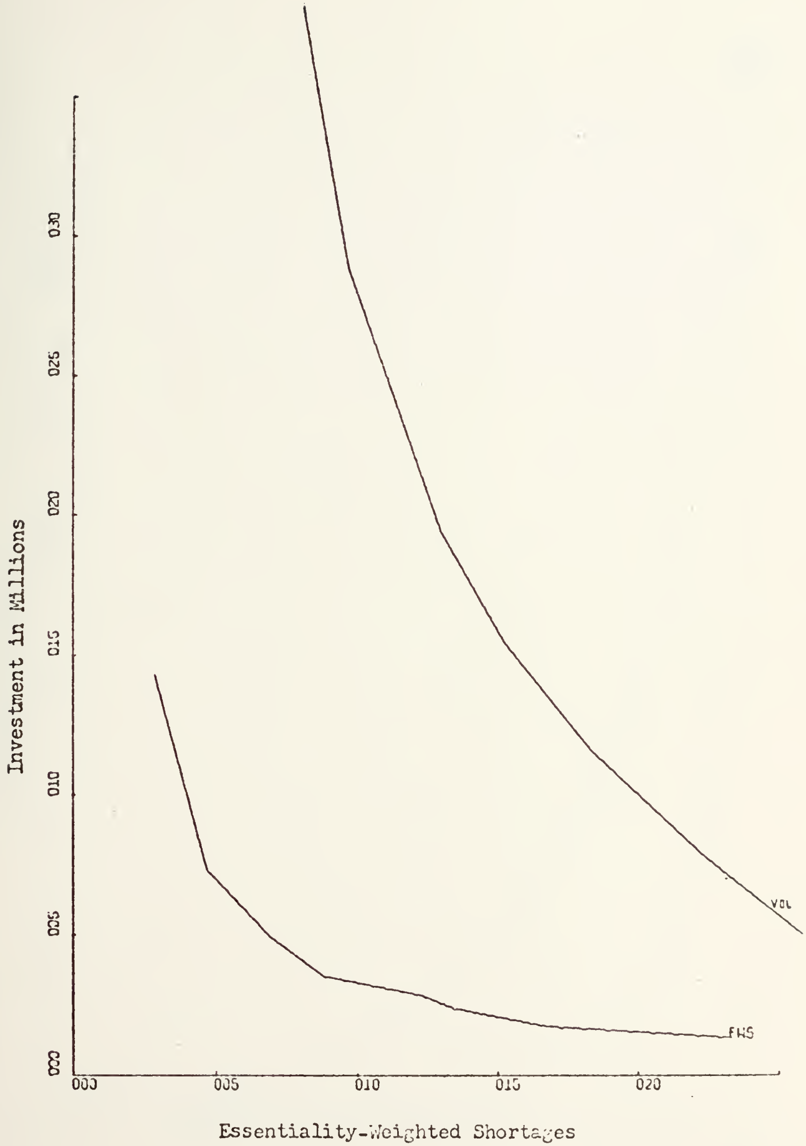

Figure 3: Investment versus Essentiality-Weighted Shortages with S' = 1

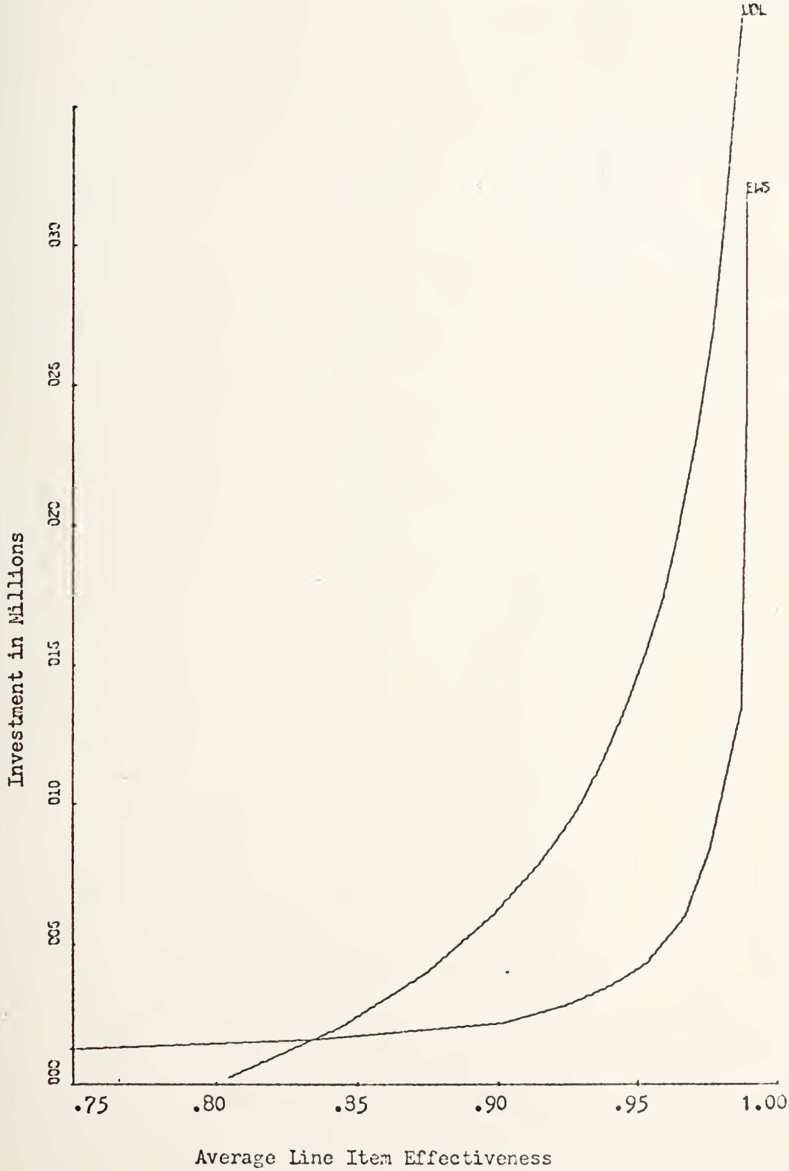

Figure 4: Investment versus Average Line Item Effectiveness with S' = 100

Figure 5: Investment versus Average Line Item Effectiveness with S' = 1

a much smaller investment. Also, for given levels of investment the EWS model provided markedly lower essentiality-weighted shortages and markedly higher line item effectiveness.

To emphasize the magnitude of the differences a few specific comparisons were made by interpolating from Figure 4. For a 90% line item effectiveness, for example, the VOL model required a $6.4 million investment compared to the $2.2 million investment required by the EWS model, a savings of $4.2 million. For a 95% line item effectiveness, the figures were $14.4 million for the VOL model and $4.1 million for the EWS model, a savings of $10.3 million. So at higher levels of effectiveness, the EWS model is increasingly better! It should also be noted that, for the EWS model to achieve 95% line item effectiveness, it required only slightly more than the value of average monthly demand in investment. By comparison, at the same level of effectiveness the VOL model required more than 3.75 times the value of average monthly demand in investment.

Additional comparisons were made at fixed investment levels. At an investment of $4 million, the EWS model produced 94.9% line item effectiveness versus 87.6% for the VOL model. At $8 million, the EWS model produced 97.8% line item effectiveness compared to 91.6% for the VOL model.

Table 6 shows the performance of the EWS model for various values of S' with all other parameters fixed. Figure 6 displays graphically the increased level of protection (in terms of line item effectiveness) afforded

high-essentiality items by the EWS model for various values of S'. With S' at 20 or more, line item effectiveness for these items was above 98%. In this particular case, MINR was set at 0.01; the maximum attainable line item effectiveness was thus 99%. For smaller values of MINR, it is anticipated (on the basis of additional tests) that line item effectiveness for these items would continue to increase for values of S' greater than 20. In any case, the performance of the EWS model with respect to high-essentiality items should be contrasted with the performance of the VOL model, which, as expected, tended to provide the same level of protection for both high-essentiality and non-high-essentiality items (see Tables 2 and 3).

Table 6. Performance of the EWS model at various levels of S'.

S'	INV	EWS	LIE	LIEHE
1	3.429	9.79	.9433	.9436
10	3.714	13.11	.9470	.9799
20	3.801	16.69	.9474	.9841
40	3.863	24.16	.9475	.9849
60	3.887	31.72	.9475	.9849
80	3.901	39.11	.9475	.9852
100	3.914	46.48	.9475	.9855
120	3.927	53.93	.9476	.9861
200	4.226	83.55	.9477	.9870

θ: .005
MINR: .01
MAXR: .50

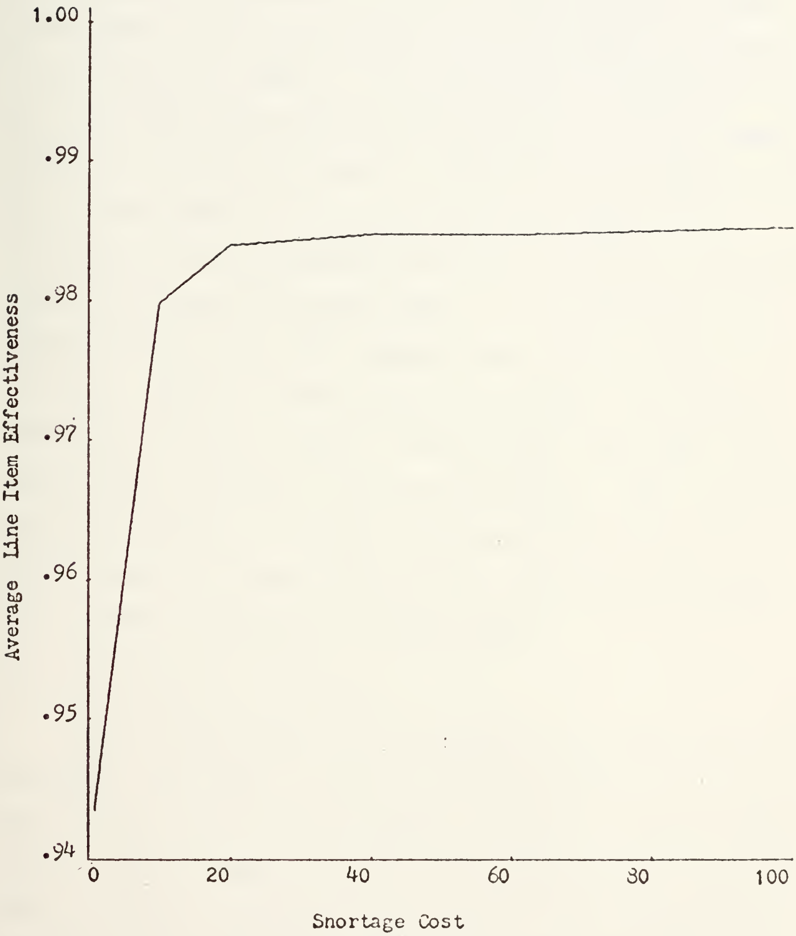

Figure 6: Line Item Effectiveness for High-Essentiality Items vs. Shortage Cost

The VOL model was designed to make a trade-off between investment level and the number of resupply requisitions generated per month [3]. The EWS model, on the other hand, was designed to optimize on effectiveness within a budget constraint, but without considering the number of resupply requisitions per month. It is not surprising therefore that the VOL model consistently generated fewer resupply requisitions than did the EWS model. The difference tended to lie in the range 350 to 375 requisitions per month. This difference must be weighed against the savings in investment dollars afforded by the EWS model. At the 95% line-item effectiveness level, for example, the savings is $10.3 million. An AFS has the capacity to produce and process additional resupply orders at very little additional cost because the procedure is largely automated. Thus, whatever savings in terms of resupply order processing is provided by the VOL model, it would be more than offset by the savings in investment afforded by the EWS model.

C. SENSITIVITY OF EWS MODEL

The EWS model was tested extensively for sensitivity to parameter variations. Figure 7 was plotted from the data in Table 4 and shows the relationship between θ and investment level. Table 7 contains test results for various values of MAXR with other parameters held fixed. Figure 8 shows the effect on the investment level of varying MAXR between 0.15

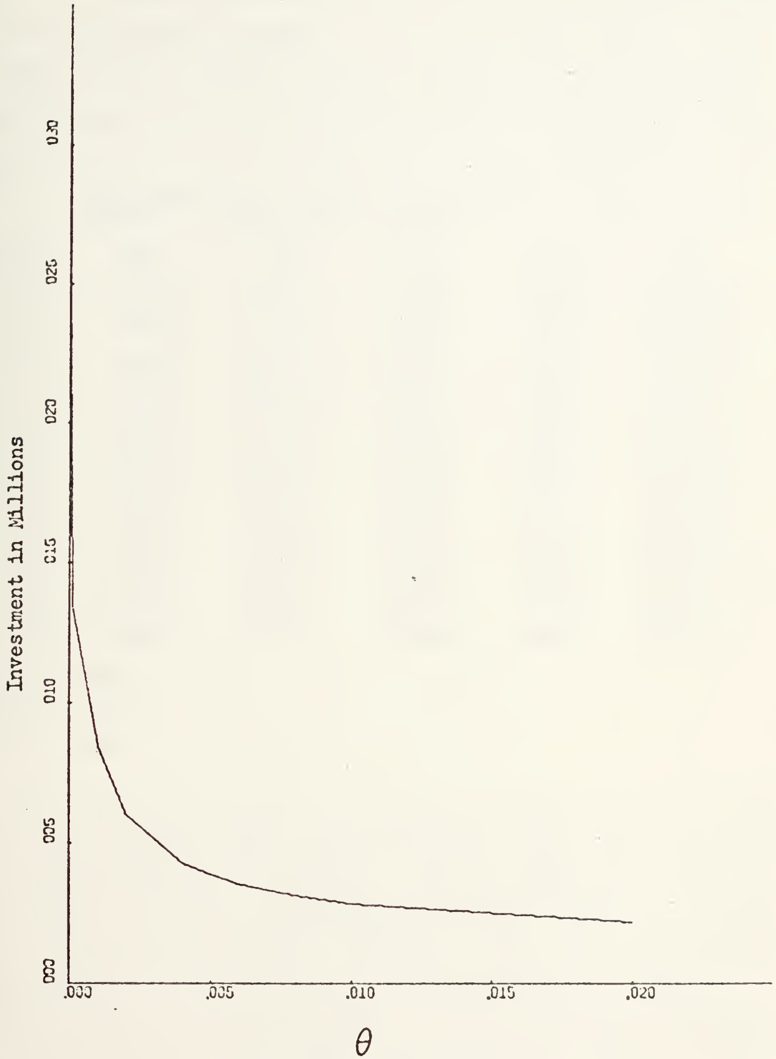

Figure 7: Investment versus θ

and 1.0. Figure 9 shows the effect on line item effectiveness of the same changes in MAXR. It is seen that smaller values of MAXR provide slightly higher levels of overall line item effectiveness, but at considerable cost in investment dollars.

Table 7. Performance of the EWS for various values of MAXR when S' = 100 and θ = 0.005.

MAXR	INV	EWS	LIE	LIEHE	R/M
0.15	9.538	45.80	.9608	.9868	933.50
0.20	7.800	46.22	.9558	.9864	925.67
0.25	6.629	46.34	.9528	.9856	913.00
0.30	5.796	46.40	.9507	.9855	907.75
0.35	5.204	46.42	.9496	.9855	904.75
0.40	4.708	46.46	.9486	.9855	899.50
0.45	4.283	46.47	.9480	.9855	897.83
0.50	3.914	46.48	.9475	.9855	895.04
0.60	3.343	46.49	.9469	.9855	891.67
0.75	3.083	46.49	.9466	.9855	888.46
1.00	2.935	46.50	.9464	.9855	884.21

θ: .005
MINR: .01
S': 100

The effect of changes in the high-essentiality shortage cost S' has already been examined above. Tests were also run on the effect of varying the value of MINR; the results (not shown) indicated that MINR has minimal impact on investment, essentiality-weighted shortages, and overall line-item effectiveness. The effect on high-essentiality line item effectiveness was discussed above. It is therefore recommended that MINR be set at some small, positive value such as 0.001.

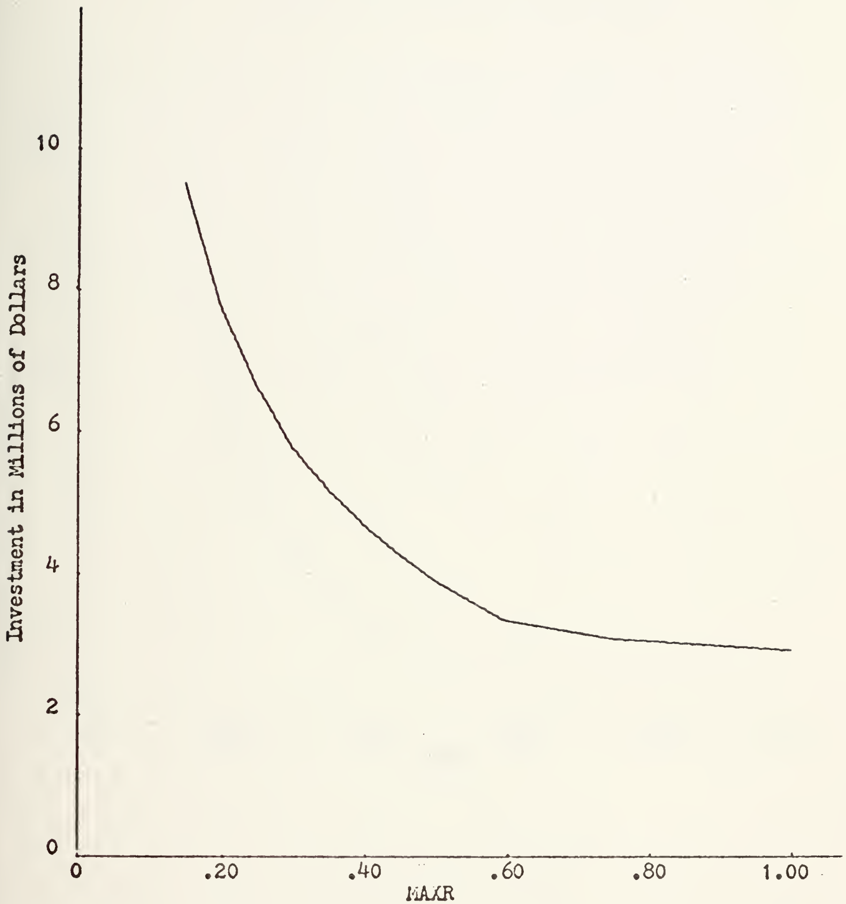

Figure 8: Investment versus MAXR

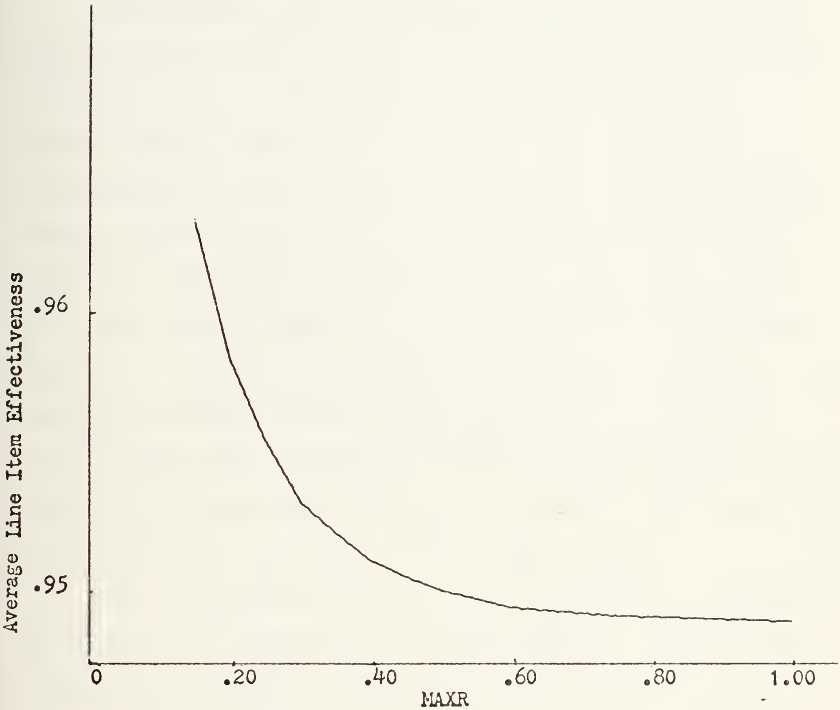

Figure 9: Average Line Item Effectiveness versus MAXR

V. CONCLUSIONS AND RECOMMENDATIONS

The inventory model presented in Chapter III of this thesis (the EWS model) appears to be superior to the model now being used by Combat Stores Ships (the VOL model). The EWS model is designed to operate in an environment in which resupply orders may be placed only at specified times, whereas the VOL model assumes that orders may be placed at any time. The EWS model explicitly considers the presence of a constraint on total investment, whereas the VOL model does not. The EWS model explicitly considers the form of the statistical distribution of demand; the VOL model cannot. In each case, the assumptions underlying the EWS model conform to the conditions which exist in the AFS environment; the assumptions underlying the VOL model do not.

From a practical point of view, the results of Chapter IV confirm the superiority of the EWS model. The EWS model allowed system operation to take place at a much lower investment level for a given performance level and provided much better performance for a given investment level. At the 95% line item effectiveness level, for example, the proposed model required less than one third as many investment dollars than did the VOL model.

A rather important by-product of this analysis is the discovery that the distribution of inventory demand experienced by Combat Stores Ships can be closely approxi-

mated by the compound Bernoulli/Exponential distribution described by equation (A-1) of Appendix A. It is recommended that inventory data from other sources be tested for conformity to this distribution. If the results of such tests are affirmative, the implications in the field of inventory theory will be far-reaching.

In view of the huge potential savings in investment levels involved, it is recommended that the Naval Supply Command give serious consideration to the implementation in Combat Stores Ships of the inventory model proposed herein. Implementation would require the reprogramming of only one segment of the Shipboard Uniform Automated Data Processing System (SUADPS-207), the levels computation segment. The revised resupply decision rule could be implemented simply by setting all reorder points to one less than the corresponding requisitioning objectives. It is anticipated that the cost of reprogramming would be more than offset by the savings in investment in a relatively short period of time.

APPENDIX A. THE DISTRIBUTION OF DEMAND

Demand is stochastic in nature; that is, demand in a future period cannot be forecast with certainty. This does not mean, however, that probability statements about the value of future demand cannot be made. If enough is known about the statistical distribution of demand, statements can be made, for example, about the probability that demand in a future period will exceed a given level.

The event that demand (D) exceeds available supply (x) has been defined in Chapter II as a stockout, and the probability of a stockout occurring when x units of an item are stocked was defined as the stockout risk associated with stocking x units of that item. This may be expressed more compactly as $Pr(D>x) = \text{risk}$, or $\overline{F}(x) = \text{risk}$, where $\overline{F}(x)$ is called the complementary cumulative distribution function of D evaluated at x.

If the form of $\overline{F}(x)$ is known, this knowledge may be used to determine the stock level x necessary to obtain a desired level of risk. The purpose of this appendix is therefore to describe the form of $\overline{F}(x)$ for the case of AFS demand.

Demand history records of two Pacific Fleet AFS's, USS MARS (AFS-1) and USS WHITE PLAINS (AFS-4), were obtained from FMSO. These records covered a period of 24 calendar months, ending in February, 1975. These two

ships were selected because during this period they were both homeported in Sasebo, Japan, and therefore spent the entire period in the Western Pacific area. They operated on an alternating basis during this period (i.e., when one was operating, the other was not). Their demand records were therefore aggregated (as if they had come from one AFS operating continously rather than two AFS's alternating). Underway replenishment operations during this period were conducted throughout the Western Pacific area; the AFS's reloaded, as was mentioned in the introduction, at NSD Subic.

From these data, monthly demand records for 2758 (out of approximately 11,000) line items were selected using the criterion that average monthly demand must be greater than 1.0. A sample of 250 items (taking every tenth item) was taken from this population of 2758 items for purposes of analysis. For this sample, the average number of months with non-zero demand per line item was 7.62 (out of 24 monthly observations). The empirical distribution of this non-zero variable aggregated over the 250 items is shown as Figure 10.

One striking characteristic of the sample was the high incidence of zero observations. It became readily apparent that any appropriate probabilistic model of demand would have to account for a positive probability mass at zero.

Frequency (Number of Line Items)

N		
1	xx	2
2	xxxxxx	6
3	xxxxxxxxxxxxxxxxxxxxxxxx	24
4	xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx	38
5	xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx	32
6	xxxxxxxxxxxxxxxxxxxxxxxxxx	26
7	xxxxxxxxxxxxxxxxxxxxxxxx	24
8	xxxxxxxxxxxxxxxxxxxxx	21
9	xxxxxxxxxxxx	12
10	xxxxxxxxxx	10
11	xxxxxxxxxxx	11
12	xxxxxx	6
13	xxxxxxxxxx	10
14	xx	2
15	xxxxxxxxx	9
16	xxxxxxx	7
17	xxxx	4
18	x	1
19	xx	2
20		0
21	xxx	3
22		0
23	x	1
24		0

N = Number of non-zero observations per line item (out of 24)

Figure 10: Observed Distribution of the Number of Non-Zero Demand Observations per Line Item

Subjective examination of individual demand records seemed to indicate that the high incidence of zero observations was not restricted to low-demand items, as might be expected. This suggested that the number of non-zero observations for a given line item was unrelated to the average value of those non-zero observations. In order to test this hypothesis, a sub-sample of 25 items was taken. A scatter plot (not shown) failed to show any apparent relationship. This was confirmed by computing the Spearman rank-order correlation for the sample and testing it for significance. The value of the correlation coefficient obtained was -0.12, which was found to be not significantly different from zero at the .05 level of significance (with 23 degrees of freedom).

This suggested that the demand process may be comprised of two unrelated subprocesses, with one process determining whether a demand will occur and the other determining the quantity of the demand, given that it does occur. The former process lends itself to modelling as a Bernoulli process with parameter p; p being the probability that a demand does occur. The latter process appeared on the basis of some preliminary exploratory data analyses to approximate an exponential distribution. A probability function resulting from the mixing of a Bernoulli process and an exponential distribution is shown in equation (A-1).

$$f(x) = \begin{cases} 1-p & , x = 0 \\ p\lambda e^{-\lambda x}, & x > 0, \end{cases} \tag{A-1}$$

where $1/\lambda$ is the expected value of demand, given that demand is greater than zero. The corresponding complementary cumulative distribution function is given by (A-2).

$$\bar{F}(x) = pe^{-\lambda x}, \ x \geqq 0. \tag{A-2}$$

It was hypothesized that demand is distributed according to the distribution described above. This is statistically equivalent to the hypothesis that the conditional distribution of demand, given that a demand occurs, is exponential. In order to test this latter hypothesis, Kolmogorov-Smirnov tests for goodness of fit were performed on demand data for each of the 250 line items in the sample. To compensate for the fact that the parameter λ was estimated from the data, tables by Lilliefors [7] were used. At the .05 level of significance, the hypothesis was accepted 229 times, or 91.6% of the time. At the .10 level, the hypothesis was accepted 223 times, or 89.2% of the time.

The above goodness-of-fit tests by themselves provide strong evidence of the applicability of the exponential distribution as a model for the conditional distribution of positive demand. Additional tests were performed, however, to further evaluate the goodness of fit in the region of primary concern in inventory models, the upper or right-hand tail of the distribution. The tests were

performed using four additional mutually-exclusive samples of 250 items each and were intended to provide cross-validation of the data as well as confirmation of the previous result.

The procedure for these tests was as follows: With the first sample, the theoretical 80th percentile of positive demand was calculated for each item, assuming that positive demand is distributed exponentially. This is simply the value of x such that $e^{-\lambda x} = .20$. The actual percentage of non-zero observations greater than x (i.e., the observed risk) was then computed for each item in the sample. If it were true that the conditional distribution of positive demand is exponential, one would expect that the average value of the observed risks over the 250 items would be close to 0.20. The hypothesis that the mean of the observed risks was 0.20 was tested using a "t" test with 249 degrees of freedom. At the .05 level of significance the observed mean was found to be not significantly different from 0.20.

The above procedure was repeated for the 85th, 90th, and 95th percentiles, each time using a different sample in order to avoid the problem of simultaneous inference. The results are summarized in Table 8. At the .05 level of significance the critical value of "t" is 1.960, and at the .01 level it is 2.576. Thus, at the .05 level only the third of the four samples was found to have failed the

test (but on the conservative side). This could simply mean that this was not a representative sample. In any case, no significant differences between the observed and theoretical means were noted at the .01 level of significance.

Table 8: Results of "t" tests on observed risks with 249 degrees of freedom.

Theoretical Risk	0.20	0.15	0.10	0.05
Mean Observed Risk	0.1916	0.1385	0.0881	0.0424
Variance of Observations	0.0117	0.0109	0.0080	0.0041
"t" Statistic	-1.2249	-1.7502	-2.0541	-1.8647

As a final indicator of goodness of fit, the computer program listed after Appendix B was run with the MAXR and MINR constraints set equal at various levels thought to be of interest. With the program thus constrained, none of the other parameters had any effect and the model was forced to compute the same theoretical risk for each of the 2758 items in the inventory (assuming the Bernoulli/exponential distribution). The mean observed risk (percentage of the time demand exceeded R_i) was then computed by taking one minus line item effectiveness. The results are displayed in Table 9. The program did not generate sufficient detail for hypothesis testing, but these results are nonetheless strong indicators of the closeness of the fit and of the validity of the model, particularly at low levels of risk. The results of Chapter IV indicate that

these low levels of risk are economically achievable on the average using the model of Chapter III, so it is the low-risk region which is of primary interest in this application.

Table 9: Additional Goodness of Fit Indicators.

Theoretical Risk	.01	.02	.04	.06	.08	.25
Mean Observed Risk	.0101	.0207	.0408	.0597	.0781	.2190

In summary, five independent samples of 250 items each from a population of 2758 have been tested for goodness of fit to the Bernoulli/exponential distribution given by equation (A-1). Several indicators computed using the entire population were also examined. Collectively the results of these tests provide strong evidence that this distribution describes the distribution of demand for any given item quite well.

APPENDIX B: ANALYSIS OF RISK ASSOCIATED WITH THE CURRENT MODEL

The setting of reorder points at a fixed number times the average monthly demand for each item is apparently an attempt to achieve the same level of protection against stockouts for each item in the inventory. The implications of such a policy with respect to stockout risk may be analyzed using the results of Appendix A. The risks computed below do not consider the protection provided by the operating level, which is of course variable.

From equation (A-2) the stockout risk associated with carrying x units of stock (of a particular item) is given by

$$\overline{F}(x) = pe^{-\lambda x}, \; x \geqq 0.$$

The mean of this distribution is p/λ. Thus n times the average monthly demand is np/λ and the associated risk is

$$\overline{F}(np/\lambda) = pe^{-\lambda(np/\lambda)} = pe^{-np}, \qquad \text{(B-1)}$$

which is seen to be independent of λ. This windfall considerably simplifies the analysis, as λ does not need to be considered. The average number of non-zero observations in the sample displayed in Figure 10 was 7.62 out of 24. Thus 7.62/24 = .3175 was the average value of p for the sample. The risks associated with stocking n times

average monthly demand for one monthly operating period can be easily computed for various values of n and p using equation (B-1); typical results are summarized in Table 10.

Examination of equation (B-1) or Table 10 emphasizes the following properties of the demand distribution. First, the risk associated with carrying no stock at all is p, not 1.0 as might be expected. This is simply because there can be no stockout if there is no demand. Secondly, for a given n, the value of risk varies with p.

Table 10: Risk associated with stocking n times average monthly demand.

n	p=.10	p=.3175	p=.50	p=.75	p=1.00
0	.1000	.3175	.5000	.7500	1.0000
1	.0905	.2311	.3033	.3543	.3679
2	.0819	.1683	.1839	.1673	.1353
3	.0741	.1225	.1116	.0790	.0498
4	.0670	.0892	.0677	.0373	.0183
5	.0607	.0649	.0410	.0176	.0067
6	.0549	.0473	.0249	.0083	.0025
7	.0497	.0344	.0151	.0039	.0009
8	.0449	.0250	.0092	.0019	.0003
9	.0407	.0182	.0056	.0009	.0001
10	.0368	.0133	.0034	.0004	.0000

The point of this analysis is that stocking the same number of months' demand for each item does <u>not</u> provide a uniform distribution of stockout risk. The associated risks vary significantly as functions of n and p. For the lower values of n (0,1,2), the lowest risks (of those

computed) are associated with p = .10. For the higher values of n, on the other hand (6 through 10), the highest risks are associated with p = .10.

```
      INTEGER*4 DEM(24),RS1,RS2
      LOGICAL*1 EI
      REAL*4  MINQ,MAXQ,MINR,MAXR,MU,LIE1,LIE2,LIEHE1,
     1  LIEHE2
  100 FORMAT (5E10.4)
  200 FORMAT (1H1,'MOE-S FOR',13X,'VOL MODEL',11X,
     1   'EWS MODEL')
  201 FORMAT (1H0,'LIE',8X,2F20.4)
  202 FORMAT (1H0,'LIE HE',5X,2F20.4)
  203 FORMAT (1H0,'EWUS',7X,2F20.2)
  204 FORMAT (1H0,'NR RQNS/MO',1X,2F20.2)
  205 FORMAT (1H0,'LINE ITEMS = ',I18)
  206 FORMAT (1H0,'INPUT PARAMETERS WERE AS FOLLOWS:')
  207 FORMAT (1H0,'THETA =',F10.6,'  MINR = ',F8.3,
     1       '  MAXR =',F8.3,'  SCOST =',F7.1)
  208 FORMAT (1H0,'OLM =',F6.1,'  MINQ =',F6.1,
     1       '  MAXQ =',F6.1,'  SL =',F6.1,'  OST =',F6.1)
  210 FORMAT (1H0,'INVESTMENT',1X,2F20.2)
      CALL ERRSET (214,300,-1,1)
    5 REWIND 3
      RS1 = 0
      RS2 = 0
      NR  = 0
      LID = 0
      LIDHE = 0
      LIS1 = 0
      LIS2 = 0
      LISHE1 = 0
      LISHE2 = 0
      EWUS1 = 0.0
      EWUS2 = 0.0
      VRO = 0.0
      VR  = 0.0
      READ (5,100) OLM,MINQ,MAXQ,SL,OST
C     NEGATIVE OLM STOPS PROGRAM, ZERO OLM DOES EWS ONLY
      IF (OLM) 70,7,7
    7 CONTINUE
      READ (5,100) THETA,MINR,MAXR,T,SCOST
C     ASSUMES 100 % RESUPPLY AT EOM
   10 READ (3,END=60) NIIN,UP,EI,NF,XBAR,Z,Z,(DEM(J),J=1,24)
      NR = NR + 1
      S  = 1.0
      IF (EI) S = SCOST
      IF (OLM) 70,21,11
   11 CONTINUE
      OL = OLM * SQRT(XBAR/UP)
      RP = (SL + OST)*XBAR
      RO = RP + OL
      VRO= VRO + RO*UP
      OH = RO
      DO 20 I = 1,24
      IF (OH - DEM(I)) 14,12,12
   12 OH = OH - DEM(I)
      GO TO 16
   14 OH = 0.0
      LIS1 = LIS1 + 1
      IF (EI) LISHE1 = LISHE1+ 1
      EWUS1 = EWUS1 + (DEM(I)-OH)*S
   16 IF (OH-RP) 18,18,20
   18 RS1 = RS1 + 1
      OH = RO
   20 CONTINUE
   21 CONTINUE
C     NOW DO EWUS MODEL
C     COMPUTE MU
      SUM = 0.0
      P   =  0.0
      DO 30 I = 1,24
      IF (DEM(I)) 30,30,22
   22 P = P + 1.0
      SUM = SUM + DEM(I)
   30 CONTINUE
```

```
      MU = SUM/P
C     COMPUTE RISK AND R
      RISK = THETA*UP/S
      IF (MAXR - RISK) 32,40,34
   32 RISK = MAXR
      GO TO 40
   34 IF (RISK - MINR) 36,40,40
   36 RISK = MINR
   40 CONTINUE
      P = P/24.0
      IF (RISK .GT. P) RISK = P
C     RISK MUST BE .LE. P BY DEFINITION
      Q = RISK/P
      R = -1.*MU*ALOG(Q)
      VR = VR + R*UP
C     OPERATE 24 MONTHS
      OH = R
      DO 50 I = 1,24
      LID = LID + 1
      IF (EI) LIDHE = LIDHE + 1
      IF (OH - DEM(I)) 44,42,42
   42 OH = OH - DEM(I)
      GO TO 46
   44 OH = 0.0
      LIS2 = LIS2 + 1
      IF (EI) LISHE2 = LISHE2 + 1
      EWUS2 = EWUS2 + (DEM(I) - OH)*S
   46 IF (R - OH) 50,50,48
   48 RS2 = RS2 + 1
      OH = R
   50 CONTINUE
      GO TO 10
C     NOW COMPUTE STATISTICS
   60 CONTINUE
      Z = LID
      LIE1 = (Z - LIS1)/Z
      LIE2 = (Z - LIS2)/Z
      Z = LIDHE
      LIEHE1 = (Z - LISHE1)/Z
      LIEHE2 = (Z - LISHE2)/Z
      WRITE (6,200)
      WRITE (6,201) LIE1,LIE2
      WRITE (6,202) LIEHE1,LIEHE2
      WRITE (6,203) EWUS1,EWUS2
      WRITE (6,210) VRO,VR
      Z1 = RS1/24.0
      Z2 = RS2/24.0
      WRITE (6,204) Z1,Z2
      WRITE (6,205) NR
      WRITE (6,206)
      WRITE (6,208) OLM,MINQ,MAXQ,SL,OST
      WRITE (6,207)  THETA,MINR,MAXR,SCOST
      GO TO 5
   70 CONTINUE
      STOP
      END
```

LIST OF REFERENCES

1. Cooper, CDR Donald R. and Bobbitt, William T., "History of Afloat Automated Supply Systems," Navy Supply Corps Newsletter, pp. 8-11, May-June 1974.

2. Gabriel, Ronald J., Fleet Material Support Office ALRAND Working Memorandum 194, Resupply Decision Rules for Combat Stores Ships, 20 February 1970.

3. Naval Supply Systems Command Manual, SUADPS-207 Support Procedures.

4. Gabriel, R. J., and Peters, E. L., Fleet Material Support Office Operations Analysis Department Report 71, Uniform AFS Resupply Decision Rules User's Manual, 2 September 1971.

5. Hadley, G., and Whitin, T. M., Analysis of Inventory Systems, Prentice-Hall, 1963.

6. Thompson, D. W., Fleet Material Support Office ALRAND Working Memorandum 240, Frequency Distributions for 1975 Pacific FIRL/FILL, 18 April 1975.

7. Lilliefors, Hubert W., "On the Kolmogorov-Smirnov Test for the Exponential Distribution with Mean Unknown," American Statistical Association Journal, v. 64, pp. 387-389, March, 1969.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
2.	Defense Logistics Studies Information Exchange Fort Lee, Virginia 23801	1
3.	Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
4.	Department Chairman, Code 55 Department of Operations Research and Administrative Sciences Naval Postgraduate School Monterey, California 93940	2
5.	Assoc. Professor A. W. McMasters, Code 55 Mg Department of Operations Research and Administrative Sciences Naval Postgraduate School Monterey, California 93940	2
6.	Professor R. R. Read, Code 55 Re Department of Operations Research and Administrative Sciences Naval Postgraduate School Monterey, California 93940	1
7.	Director, Material Division (NAVOP 41) Office of the Deputy Chief of Naval Operations (Logistics) Navy Department Washington, D.C. 20350	1
8.	Director, Systems Analysis Division (NAVOP 964) Program Planning Office Navy Department Washington, D.C. 20350	1

No. Copies

9.	Director, Afloat Systems Design Branch (NAVSUP 04522) Naval Supply Systems Command Headquarters Washington, D.C. 20376	1
10.	Mr. J. W. Prichard (NAVSUP 0411A) Naval Supply Systems Command Headquarters Washington, D.C. 20376	1
11.	LCDR R. D. Perkins, SC, USN (NAVSUP 0411I) Naval Supply Systems Command Headquarters Washington, D.C. 20376	1
12.	Mr. H. J. Lieberman (NAVSUP 0431B) Naval Supply Systems Command Headquarters Washington, D.C. 20376	1
13.	Mr. David B. Cassing, President Center for Naval Analyses 1401 Wilson Blvd. Arlington, Virginia 22209	1
14.	CAPT C. W. Rixey, SC, USN Fleet Supply Officer Commander in Chief, U. S. Atlantic Fleet Norfolk, Virginia 23511	1
15.	Fleet Supply Officer Commander in Chief, U. S. Pacific Fleet FPO San Francisco, California 96610	1
16.	CDR P. F. McNall, SC, USN (FMSO 94) Fleet Material Support Office Mechanicsburg, Pennsylvania 17055	1
17.	Mrs. Joyce Lerch (FMSO 9712) Fleet Material Support Office Mechanicsburg, Pennsylvania 17055	1
18.	LT Charles F. Taylor, Jr., SC, USN Planning Department (Code 40C) Naval Supply Center Charleston, South Carolina 29408	1
19.	CDR J. P. Dowling, SC, USN Naval War College Newport, Rhode Island 02840	1

No. Copies

20. Professor P. W. Zehna, Code 55Ze — 1
Department of Operations Research and
Administrative Sciences
Naval Postgraduate School
Monterey, California 93940

21. Assoc. Professor F. R. Richards, — 1
Code 55Rh
Department of Operations Research and
Administrative Sciences
Naval Postgraduate School
Monterey, California 93940

22. Mr. B. Rosenman, Chief — 1
Inventory Research Office
Army Material Command
Frankford Arsenal
Philadephia, Pennsylvania 19137

23. Mr. V. J. Presutti, Jr. — 1
Operations Analysis Office
Headquarters, Air Force Logistics Command
Wright-Patterson Air Force Base, Ohio
45433

24. Mr. Chantee Lewis — 1
Naval War College
Newport, Rhode Island 02830

25. RADM Wallace R. Dowd, SC, USN — 1
Commander, Naval Supply Systems Command
Washington, D. C. 20376

Taylor

A multi-item inventory model for Combat Stores ships.

Taylor

A mulit-item inventory model for Combat Stores ships.